Non-linear Transport in Quantum-Hall Smectics

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Abstract. Recent transport experiments have established that two-dimensional electron systems with high-index partial Landau level filling, $\nu^* = \nu - [\nu]$, have ground states with broken orientational symmetry. In a mean-field theory, the broken symmetry state consists of electron stripes with local filling factor $[\nu] + 1$, separated by hole stripes with filling factor $[\nu]$. We have recently developed a theory of these states in which the electron stripes are treated as one-dimensional electron systems coupled by interactions and described by using a Luttinger liquid model. Among other things, this theory predicts non-linearities of opposite sign in easy and hard direction resistivities. In this article we briefly review our theory, focusing on its predictions for the dependence of non-linear transport exponents on the separation d between the two-dimensional electron system and a co-planar screening layer.

1 Introduction

Recent transport experiments [1, 2, 3] have established that the resistivity of a two-dimensional electron system with weak disorder and valence orbital Landau level filling ν^* close to $1/2$ is anisotropic when the valence orbital Landau level index $N \geq 2$. Apparently the ground state spontaneously breaks orientational symmetry, a property believed to be associated with the uni-directional charge-density-wave ground states predicted [4, 5] in this regime by Hartree-Fock mean-field theory [7]. The charge-density-wave (CDW) state consists of electron stripes of width $a\nu^*$ with local Landau level filling factor $\nu = [\nu] + 1$, separated by hole stripes of width $a(1-\nu^*)$ with local filling factor $\nu = [\nu]$. Here a, the CDW period, is comparable to the Landau level's cyclotron orbit diameter. Because of symmetry properties shared with the smectic state of classical liquid crystals, emphasized by Fradkin and Kivelson [6], these states have been referred to as quantum Hall smectics, a practice we follow here. The most important transport property of quantum Hall smectics is that dissipation occurs over a wide range of filling factors surrounding $\nu^* = 1/2$ and is not activated at low temperatures. This behavior is *not* consistent with the properties of a CDW state, which would be pinned by the random disorder potential and have a large gap for mobile quasiparticle excitations, and suggests that, although Hartree-Fock theory hints at the energetic motivation for a ground state with broken orientational symmetry, the description which it provides of the ground state is flawed.

Several recent theoretical papers [8, 9, 10, 11, 12, 13] have addressed the properties of quantum Hall smectics and the energetic competition between CDW states, compressible composite-fermion states, and paired incompressible quantum Hall states. In one recent paper [14] we have described a theory of quantum Hall smectics which starts from the Hartree-Fock theory ground state, recognizes the electron stripes as coupled one-dimensional electron systems, and treats residual interaction and disorder terms neglected by Hartree-Fock theory using the convenient bosonization techniques of one-dimensional electron physics. The most important conclusions of this work are the following: i) the quantum Hall smectic state is *never* the ground state but instead is always unstable, for ν^* close to 1/2 likely to an anisotropic electron Wigner crystal state; ii) for $0.4 \lesssim \nu^* \lesssim 0.6$ the interaction terms responsible for the Wigner crystal can be neglected at temperatures available in a dilution fridge; and iii) weak disorder which scatters electrons from stripe to stripe, enabling hard-direction transport, leads to non-linear transport. In this article we emphasize and expand on an experimentally important prediction of this work, namely that the strength of the transport non-linearity is sensitive to the nature of the electron-electron interaction. In particular we predict that the transport non-linearity can be enhanced by placing a screening plane close to the two-dimensional electron system.

In Section II we explain our theory of transport in quantum Hall smectics and discuss how the coefficients which govern the power-law behavior of the differential resistivity for weak disorder are related to correlations in the coupled one-dimensional electron stripes. In Section III we briefly review our theory of these correlations and explain why long-range electron-electron interactions weaken transport non-linearities. In Section IV we present numerical results for the non-linear transport coefficients for a model in which interactions in the two-dimensional electron layer are screened by a metallic layer co-planar with the electron system but separated from it by a distance d. We conclude in Section V with a brief summary.

2 Anisotropic Transport Properties

Our transport theory [14] is built on a semiclassical Boltzmann-like approach; microscopic physics enters only in the calculation of scattering rates. We choose a coordinate system where the $\hat{x}$ (horizontal) direction runs along the stripes which are separated in the $\hat{y}$ (vertical) direction. We assume that the charge density wave itself is pinned and immobilized by both the edges of the sample and weak impurities which couple to the electrons within the stripes. In this case, collective sliding motion of the charge-density will be absent, and electrical transport will be dominated by single-electron scattering across and between electron stripes. An important property of electronic states in the quantum Hall regime, is the spatial separation of states which carry current in opposite directions. In the case of the electron stripes in quantum Hall smectics, the Fermi edge states which carry oppositely directed currents along the stripes (left-going and right-going)

are located on opposite sides of the stripe. Translational invariance along the $\hat{x}$ direction, allows us to use a Landau gauge where this component of wavevector $\hbar k_x$ is a good quantum number in the absence of interactions and disorder. The single-particle states at the stripe edges have velocity magnitude v_F, the Fermi velocity. In the Landau gauge, $\hat{x}$ direction momenta are related to $\hat{y}$ direction positions by $k_x = y/\ell^2$ where $\ell \equiv (\hbar c/eB)^{1/2}$ is the magnetic length, so that states on opposite sides of the same stripe differ in momenta by a $\nu^* a/\ell^2$ and the adjacent sides of neighboring stripes differ in momenta by $(1-\nu^*)a/\ell^2$.

We now summarize the basic assumptions on which our semiclassical transport theory is based and quote the expressions implied by these assumptions. For further details see Ref. [14]. We assume that in the steady state, each edge of each stripe is characterized by a local chemical potential. Translational invariance in the $\hat{y}$ direction implies that the chemical potential drops across each stripe and between any two adjacent stripes are the same, mf μ is the chemical potential drop across an electron stripe, it follows that the potential drop between stripes is $eE_y a - \mu$, where E_y is the hard-direction electric field, the field which supports a steady state transport across the stripes. The electric field in the $\hat{x}$ direction produces a semiclassical drift in momentum space which drives the system from equilibrium. We assume that disorder scattering across and between stripes, then attempts to reestablish equilibrium and that the drift and scattering processes are in balance in the steady state. The scattering currents in the hard-direction are characterized by relaxation times τ_e and τ_h respectively. The current along a stripe is analogous to the quantized current in a long narrow Hall bar and is proportional to the chemical potential difference across that stripe. Combining these ingredients leads [14] to the following expressions for the resistivities:

$$\begin{aligned}
\rho_{\rm easy} &= \frac{h}{e^2}\,\frac{1}{\tau_e([\nu]+1)^2+\tau_h[\nu]^2}\,\frac{a}{v_F}\\
\rho_{\rm hard} &= \frac{h}{e^2}\,\frac{1}{\tau_e([\nu]+1)^2+\tau_h[\nu]^2}\,\frac{v_F\tau_e\tau_h}{a}\\
\rho_{\rm hall} &= \frac{h}{e^2}\frac{1}{\tau_e([\nu]+1)^2+\tau_h[\nu]^2}\,([\nu]+1)\tau_e+[\nu]\tau_h,
\end{aligned} \tag{1}$$

where $\rho_{\rm easy} = \rho_{xx}$, $\rho_{\rm hard} = \rho_{yy}$, and $\rho_{\rm hall} = \rho_{xy}$.

For $\nu^* = 1/2$, this theory makes a parameter free prediction for the product $\rho_{easy}\rho_{hard}$ which has been confirmed experimentally [15]. In fact, as emphasized [13] by van Oppen *et al.*, this feature of our results has a greater validity than would be suggested by our assumption of largely intact electron stripes. Our main interest here however, is in expanding on our predictions [14] for non-linearities in the easy and hard direction differential resistivities. These predictions were made on the basis of a simple lowest order renormalization group scheme for handling the infrared divergence which appear when disorder terms which scatter electrons either across or between stripes are treated perturbatively. This analysis leads to

$$\frac{1}{\tau_e} \equiv \Gamma_e \sim \Gamma_e^{(0)}(V_y/E_c)^{2\Delta_e-2}$$

$$\frac{1}{\tau_h} \equiv \Gamma_h \sim \Gamma_h^{(0)} (V_y/E_c)^{2\Delta_h - 2} \tag{2}$$

where $\Gamma_e^{(0)}$ and $\Gamma_h^{(0)}$ are Golden-rule scattering rates at the characteristic microscopic energy scale E_c, Here Δ_e is [14] the scaling dimension of the operator which scatters an electron across a stripe, which we discuss in the next section, and Δ_h is the scaling dimension of the operator which scatters an electron between neighboring stripes. The values of Δ_e and Δ_h depend on correlations induced by electron-electron interaction between stripes, and are sensitive in particular to the range of the microscopic electron-electron interaction. At $\nu^* = 1/2$, the case on which we will concentrate, $\Delta_e = \Delta_h$.

Given these expressions, it follows [14] that the non-linear differential resistivity in the hard direction

$$\frac{\partial V_y}{\partial I_y} \sim I_y^{\alpha}, \tag{3}$$

with an exponent $\alpha = 2(1-\Delta_e)/(2\Delta_e - 1)$. Similar considerations apply for the easy direction current:

$$\frac{\partial V_x}{\partial I_x} \sim I_x^{\beta} \tag{4}$$

with an exponent $\beta = 2(\Delta_e - 1)$. In the next section we show that both α and β increase when the distance to the screening plane is comparable to or smaller than the CDW period.

3 Quantum Smectic Model

The CDW state of Hartree-Fock theory [4, 5] is a single-Slater-determinant. In the valence Landau level, groups of Landau-gauge single-particle states with adjacent k_x (adjacent $\hat{y}$) are occupied to form stripes and separated by groups which are unoccupied. Small fluctuations in the positions and shapes of the stripes can be described in terms of particle-hole excitations near the stripe edges. The residual electron-electron interaction terms which scatter into these low energy states are ignored in Hartree-Fock theory and fall into two classes: "forward" scattering interactions which conserve the number of electrons on each edge of every stripe, and "backward" scattering processes which do not. The latter processes involve large momentum transfer and are unimportant [14] at accessible temperatures for ν^* near 1/2. The quantum smectic model [14], briefly described in this section includes forward scattering only. The interactions are bilinear in the 1D electron density contributions from a particular edge of a particular stripe: $\rho_{n\alpha}(x)$, with $\alpha = \pm$. Since the density of a single filled Landau level is $(2\pi\ell^2)^{-1}$, the displacement of an edge is related to its associated charge density contribution by $u_{n\alpha}(x) = \alpha 2\pi\ell^2 \rho_{n\alpha}(x)$. The quadratic Hamiltonian which describes the *classical* energetics for small fluctuations has the following general form:

$$H_0 = \frac{1}{2\ell^2} \int_{x,x'} \sum_{n,n'} u_{n\alpha}(x) D_{\alpha\beta}(x-x'; n-n') u_{n'\beta}(x')$$

$$= \frac{1}{2\ell^2} \int_{\mathbf{q}} u_\alpha(-\mathbf{q}) D_{\alpha\beta}(\mathbf{q}) u_\beta(\mathbf{q}), \tag{5}$$

where $\int_{\mathbf{q}} \equiv \int d^2\mathbf{q}/(2\pi)^2$. Here the q_y integral is over the interval $(-\pi/a, \pi/a)$ and a high momentum cutoff $\Lambda \sim 1/\ell$ is implicit for q_x.

Symmetry considerations constrain the form of the elastic kernel. In position space, the kernel must be real and symmetric so that $D_{\alpha\beta}(\mathbf{q}) = D^*_{\alpha\beta}(-\mathbf{q}) = D^*_{\beta\alpha}(\mathbf{q})$. This implies $D_{-+}(\mathbf{q}) = D^*_{+-}(\mathbf{q})$ and $\mathrm{Im}D_{\alpha\alpha}(\mathbf{q}) = 0$. Parity invariance (under $x, n, + \leftrightarrow -x, -n, -$), implies moreover $D_{++}(\mathbf{q}) = D_{--}(\mathbf{q})$. Thus, the elastic kernel is fully specified by one real function, $D_{++}(\mathbf{q})$, and one complex function, $D_{+-}(\mathbf{q})$. It will be important for our present interest that the Hamiltonian must be invariant under: $u_{n\alpha}(x) \to u_{n\alpha}(x) + const.$ For short-range interactions this implies that at long wavelengths

$$D(q_x = 0, q_y) = K_y q_y^2 + ..., \tag{6}$$

characteristic of classical smectic elasticity. As we will discuss, this conclusion must be modified in the case of long-range interactions.

A *quantum* theory of the Quantum-Hall smectic [14] is obtained by imposing Kac-Moody commutation relations on the chiral densities:

$$[\rho_{n\alpha}(x), \rho_{n'\beta}(x')] = \frac{i}{2\pi} \alpha \delta_{\alpha,\beta} \delta_{n,n'} \partial_x \delta(x - x'). \tag{7}$$

Together with Eq.(5), this relationship fully specifies the quantum dynamics. Electron operators in the chiral edge modes are related to the 1D densities via the usual bosonic phase fields: $\psi_{n\alpha} \sim e^{i\phi_{n\alpha}}$ with $\rho_{n\alpha} = \alpha \partial_x \phi_{n\alpha}/2\pi$.

Quantum properties of the smectic can be computed from the imaginary-time action,

$$\begin{aligned} S_0 &= \int_{x,\tau} \frac{1}{4\pi} \sum_{n,\alpha} i\alpha \partial_\tau \phi_{n,\alpha}\, \partial_x \phi_{n,\alpha} + \int_\tau H_0 \\ &= \frac{1}{2} \int_{\mathbf{q},\omega} \phi_\alpha(-\mathbf{q}, -\omega) M_{\alpha,\beta}(\mathbf{q}, \omega) \phi_\beta(\mathbf{q}, \omega), \end{aligned} \tag{8}$$

where in an obvious matrix notation,

$$\mathbf{M}(\mathbf{q}, \omega) = (i\omega q_x/2\pi)\sigma^z + (q_x \ell)^2 \mathbf{D}(\mathbf{q}). \tag{9}$$

Correlation functions follow from Wick's theorem and the momentum space correlator $\langle \phi_\alpha \phi_\beta \rangle = \mathbf{M}^{-1}$ with

$$\mathbf{M}^{-1}(\mathbf{q}, \omega) = \sigma_z \mathbf{M}(\mathbf{q}, -\omega) \sigma_z / \mathrm{det}\mathbf{M}(\mathbf{q}, \omega). \tag{10}$$

The effect of weak disorder on transport in quantum Hall smectics depends sensitively on the elastic constants *at* $q_x = 0$. In this limit the relevant excited states are simply Slater determinants with straight stripe edges displaced from those of the Hartree-Fock theory ground state. By evaluating the expectation

value of the microscopic Hamiltonian in a state with arbitrary stripe edge locations we find that

$$D_{\alpha\beta}(q_x = 0, q_y) = \delta_{\alpha\beta} D_0 + \alpha\beta \frac{a}{4\pi^2 \ell^2} \sum_n e^{i q_y a n} \Gamma(y^0_{n\alpha} - y^0_{0\beta}), \tag{11}$$

where the constant D_0 is such that the condition $\sum_{\alpha\beta} D_{\alpha\beta}(\mathbf{q} = 0) = 0$, and the positions $y^0_{n\pm} = a(n \pm \nu^*/2)$ are the ground state stripe edge locations. Here, $\Gamma(y)$ is the interaction potential between two electrons located in guiding center states a distance y apart:

$$\Gamma(y) = U(0, y/\ell^2) - U(y/\ell^2, 0), \tag{12}$$

$$U(q,k) = \int \frac{dp}{2\pi} e^{-(q^2+p^2)\ell^2/2} \, V^N_{\rm eff}(q,p) e^{-ipk\ell^2}. \tag{13}$$

The two terms in Eq. (12) are direct and exchange contributions. In Eq. (13), $V^N_{\rm eff}(q,p)$ is the Fourier transform of the effective 2D electron interaction which incorporates form-factors [11] dependent on the Landau level index. N and the ground subband wavefunction of the host semiconductor heterojunction or quantum well. The smectic states have relatively long periods proportional to the cyclotron orbit radii. Explicit calculations [4, 10, 11] show that $a \gtrsim 6\ell$ for $N = 2$. It follows that the exchange contribution to $\Gamma(y)$ is small and that $\Gamma(y)$ decreases with stripe separation in the relevant range. In this paper we address the influence of a metallic screening plane which cuts off this interaction at large distances. For $y \lesssim d$, $\Gamma(y) \sim 2e^2 \ln(d/y)$, decreasing extremely slowly with y. For separation y larger than the distance d to the screening plane, $\Gamma(y) \sim y^{-2}$, making the sum over n in Eq. 11 convergent.

4 Screening Dependence of Scaling Dimensions

The scaling dimension, Δ_e, of the operator $e^{i(\phi_{n,+} - \phi_{n,-})}$ which scatters an electron across the n-th stripe is readily evaluated from Eq.(9). We find that

$$\Delta_e = \int_{-\pi}^{\pi} \frac{d(qa)}{2\pi} W(q_x = 0, q). \tag{14}$$

Here, W is the weight function,

$$W(\mathbf{q}) = \frac{[D_{++}(\mathbf{q}) + {\rm Re} D_{+-}(\mathbf{q})]}{[D^2_{++}(\mathbf{q}) - |D_{+-}(\mathbf{q})|^2]^{1/2}}. \tag{15}$$

For 1D non-interacting electrons $\Delta_e = 1$, so that disorder is relevant and eventually leads to localization. As discussed below, $\Delta_e < 1$ for quantum Hall smectics. Disorder is even more relevant than in the non-interacting electron case. Nevertheless, since the samples in which the quantum Hall smectic is observed are of extremely high quality, there should be a wide range of temperature over which its effects can be treated perturbatively. If $\Delta_e = 1$ both hard direction

(α) and easy direction (β) non-linear transport exponents vanish. We see from Eq.(15) that $\Delta_e = 1$ if the average value of $W(\mathbf{q_y})$ is one. To understand the dependence of Δ_e on screening, we have to understand the dependence of $W(q_y)$ on both wavevector and d.

Note that W is smaller than one, increasing the relevance of disorder, when D_{++} and ReD_{+-} are opposite in sign and similar in magnitude. For each q_y in Eq. 14 the weighting factors are like those which enter in the calculation of the scaling dimension of the operator which describes backscattering from disorder in an isolated one-dimensional electron system. In continuum 1D models, D_{++} has a contribution, proportional to the Fermi velocity, from the band energy and a contribution proportional to the interaction between electrons traveling in the same direction, while $-D_{+-}$ has only an interaction contribution. For a continuum model, the effective interactions between electrons traveling in different directions is the same as that between electrons traveling in the same directions. When the interaction term is much larger than the band term D_{++} and D_{+-} are opposite in sign and nearly equal in magnitude and W is very small. This is what happens, for example, for a 1D electron system in which long-range makes the Coulomb interaction very strong at long wavelengths. When W is small, the 1D electron system is very close [16] to an electron Wigner crystal, and disorder is very strongly relevant. On the other hand when D_{+-} is much smaller than D_{++} we have a situation analogous to that in a very weakly interacting Fermion system, in which disorder is relevant but the resistivity is linear when disorder can be treated perturbatively.

With this in mind we turn to a discussion of the quantum smectic, limiting our attention to the case $\nu^* = 1/2$. Useful insight comes from examining the value of W at the end points of the integration interval, $q_y a = 0$ and $q_y a = \pi$ where both D_{++} and D_{+-} are real. For $q_y = 0$, invariance under a uniform translation of the smectic implies that $D_{++} + D_{+-} = 0$, so that $W(q_y = 0) = 0$. When all the electron stripes move together, the energetics is precisely like that of a single 1D system. In the quantum Hall regime, there is no band energy, only interaction contributions from electrons traveling in the same direction, which appear in D_{++}, and interaction contributions from electrons traveling in opposite directions, which appear in D_{+-}. The absence of a band contribution means that, for $q_y = 0$, W vanishes independent of the interaction's strength or range. When $q_y a = \pi$, on the other hand, one has

$$D_{+-}(q_y a = \pi) = \sum_n \frac{(-1)^n a}{4\pi^2 \ell^2} [\Gamma(an + a(1-\nu^*)) - \Gamma(an + a\nu^*)]. \qquad (16)$$

We see that $D_{+-}(q_y a = \pi)$ vanishes because the interaction between the top of one stripe and the bottom of the same stripe is identical to its interaction with the bottom of the next stripe up. The interactions between oppositely directed electrons effectively cancel out, and we obtain $W = 1$, just as we would for a non-interacting 1D system. Thus Δ_e is determined by the average over q_y of a weighting factor which interpolates between that of a 1D electron Wigner crystal at $q_y a = 0$ and that of a non-interacting 1D electron system at $q_y a = \pi$.

The average value of W is determined by the rate at which W goes from its $q_y = 0$ limit to its $q_y = \pi$ limit. To obtain insight into what controls this, we consider first the limit of short-range interactions. Since $\Gamma(y = 0)$ vanishes due to the cancellations of its direct and exchange contributions, the short-range limit is obtained by taking only $\Gamma(a/2) \equiv \Gamma^* \neq 0$. In this case, it follows from Eq.(11) that $(4\pi^2\ell^2)D_{+-}(q)/a = -\Gamma^*(1+\exp(-iqa))$ and that $(4\pi^2\ell^2 D_{++}(q)/a = 2\Gamma^*$, and therefore that $W(q) = |sin(qa/2)|$. The numerator of Eq.(15) vanishes like q^2 for $q \to 0$, and the denominator, which is proportional to the collective mode velocity, vanishes like $|q|$. Note that the expression for $W(q)$ is independent of Γ^*. The average of $W(q)$ may be evaluated analytically in this case, and we obtain for short-range interactions $\Delta_e = 2/\pi \approx 0.6366$.

For the realistic case, analytic calculations are no longer possible, but the behavior of W can be simplified, at least at small q if the exchange contribution to $\Gamma(y)$ is neglected in constructing $D_{\alpha,\beta}(q)$. In this case $\Gamma(y)$ is the simply the 1D transform to coordinate space of the reciprocal space interaction $U(p) \equiv \exp(-p^2\ell^2/2)V^N_{eff}(0,p)$, i.e. it is the Coulomb interaction between lines of charged smeared by N-dependent cyclotron orbit form factors. The components of $D_{\alpha,\beta}(q)$, are then Fourier transforms back to reciprocal space, but with additional 'umklapp' terms because this transform is discrete. We find that

$$D_{+-}(q) = \frac{-1}{4\pi^2\ell^2}\exp(-iqa/2)\sum_{j=-\infty}^{\infty}(-)^j U(q+2\pi j/a)$$
$$D_{++}(q) = D_0 + \frac{1}{4\pi^2\ell^2}\sum_{j=-\infty}^{\infty} U(q+2\pi j/a) \tag{17}$$

and

$$D_0 = \frac{1}{4\pi^2\ell^2}\sum_{j=-\infty}^{\infty} U(2\pi j/a)[(-)^j - 1]. \tag{18}$$

For the case of a Coulomb interaction, $U(q) = [2\pi e^2(1-\exp(-2qd))]/q$ where d is the distance to a screening plane described by an image charge model. For large or infinite d, the $j = 0$ terms dominate the sums in Eqs. (17). The above expression for $U(q)$ applies when $q\ell \lesssim 1$, and therefore is always valid for the $j = 0$ terms in Eqs.(17). Note that both $D_{++}(q)$ and $D_{\pm}(q)$ are proportional to d for large d and diverge for $d \to \infty$. On the other hand D_0 remains finite for $d \to \infty$ because the $j = 0$ term is excluded from this sum. We emphasize that the numerical calculations whose results are shown below include the exchange contributions neglected in deriving Eqs.(17), and important at any value of a when d is not large.

We now examine the large d, small q behavior of the numerator and denominator of Eq.(15). For the numerator

$$D_{++}(q) + \mathrm{Re}D_{+-}(q) = \frac{1}{4\pi^2\ell^2}U(q)(1-cos(qa/2)) \sim \frac{e^2}{2\pi\ell^2}q^2da^2/2, \tag{19}$$

and for the denominator

$$D_{++}(q)^2 - |D_{+-}(q)|^2 \propto U(q) \propto [d]\ [q^2a^2]. \tag{20}$$

The first factor is square brackets in Eq.(20) comes from $D_{++}(q)+|D_{+-}(q)|$ for which the $j=0$ term dominates. The second factor is proportional to $D_{++}(q)-|D_{+-}(q)|$ for which only odd j terms survive, implying no dependence on d and analytic dependence on q. These formulas apply for $qd<1$; for $d\to\infty$, the small q behavior is obtained by replacing $U(q=0)=4\pi e^2 d$ by $U(q\to 0)=2\pi e^2/q$, *i.e.*, by replacing d by $1/2q$. We plot the square of the denominator, proportional to the square of the collective excitation velocity for several values of the screening length d in Fig. [1]. The velocity increases as d increases as expected. For $d=100\ell$, the quadratic small q behavior predicted in Eq.(20), applies only for $qa \lesssim 0.02$, and is not apparent in the plot. Instead we see the long range interaction behavior, in which the velocity is proportional to $q^{1/2}$. As is apparent from Eqs.(17), screening is irrelevant except very close to $q=0$

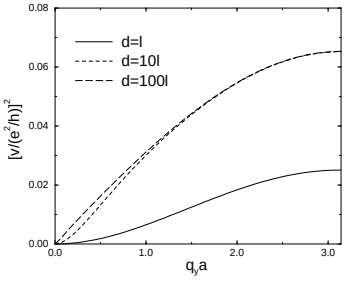

Fig. 1. Square of the collective excitation velocity (in units of $e^2/(2\pi\epsilon\hbar)$) in the $\hat{x}$ direction as a function of q_y for $d=\ell$, $d=10\ell$ and $d=100\ell$. These calculations are for $N=2$ and CDW period $a=5.8\ell$. For the dielectric function of GaAs, this velocity unit has the value 4.8×10^4m/s.

when d is large. Fig. [1] shows that once $d \gtrsim a$, the large d no-screening limit is approached. In Fig. [2] we show the weight functions for $d=\ell$, $d=10\ell$ and $d=100\ell$. At each value of d, the denominator of the weight function at small

q is proportional to $d^{1/2}|q|$ and the numerator proportional to dq^2. The weight function is therefore proportional to $|q|$ with a coefficient which varies as $d^{1/2}$. A larger value of d (less screening), leads to a weight function which increases more rapidly with q and a scaling dimension for the scattering vertex which is closer to one. In the limit of unscreened interactions, which applies down to small q for $d = 100\ell$, $W(q) \propto |q|^{1/2}$

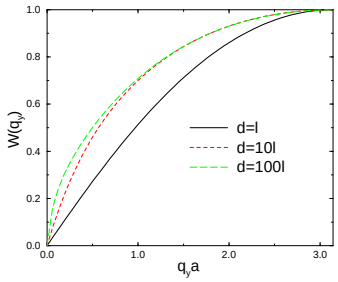

Fig. 2. Weight function *vs.* $q_y a$ for $d = \ell$, $d = 10\ell$ and $d = 100\ell$. These calculations are for $N = 2$ and CDW period $a = 5.8\ell$.

In Fig. [3] we show the dependence of the scaling dimension Δ_e and the non-linear transport exponents α and β on the distance d to the screening plan. For $d \to 0$ the numerical result is very close to that from the analytic short-range interaction model described above which leads to $\Delta_e = 2/\pi$. For $d \to 0$, the scaling dimension approaches $\Delta_e = 0.772$, a value we have been able to obtain only numerically. These relatively modest changes in the scaling dimension translate into relatively large changes in the transport exponents, particularly in α which characterizes the hard-axis non-linearity. We predict that these non-linearities will be much stronger if a screening placed in close proximity to the electron layer.

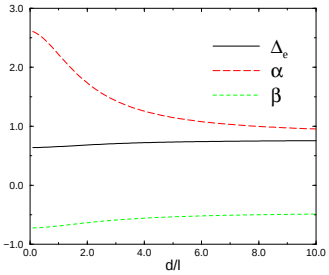

Fig. 3. Scaling dimension Δ_e and non-linear transport exponents α and β for $d = \ell$, $d = 10\ell$ and $d = 100\ell$. These calculations are for $N = 2$ and CDW period $a = 5.8\ell$.

5 Summary

Recent experiments [1, 2, 3] have established a consistent set of transport properties for high-mobility two-dimensional electron systems with high orbital index ($N \geq 2$) partially filled Landau levels. These properties differ qualitatively from those which occur in the low orbital index ($N \leq 1$) fractional quantum Hall effect regime. At large N, the dissipative resistivities are large, strongly anisotropic, and non-linear for $0.4 \lesssim \nu - [\nu] \lesssim 0.6$ within each Landau level. This anisotropic transport regime is bracketed by regions of reentrant integer quantum Hall plateaus. We have recently [14] developed a theory which is able to account for most features of these experiments. An important prediction of the theory is that the easy and hard direction resistivities should have non-integral power-law temperature dependences. In this article we have briefly summarized the theory and elaborated on its predictions for the dependence of these exponents on the distance d between the two-dimensional electron layer and a remote screening plane, predictions which are summarized in Fig. [3]. We find that Δ_e approaches two different values, both smaller than one, for $d \to 0$ and $d \to \infty$, and interpolates smoothly between these limits at finite values of d. Verification of our prediction that non-linear transport can be enhanced by introducing a screening plane and reducing d, would help substantiate our theory of quantum Hall smectics.

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